MI JUNE 13

1. Particle P has mass 3 kg and particle Q has mass m kg. The particles are m… directions along a smooth horizontal plane when they collide directly. Imme… the collision, the speed of P is 4 m s^{-1} and the speed of Q is 3 m s^{-1}. In the c… direction of motion of P is unchanged and the direction of motion of Q is … Immediately after the collision, the speed of P is 1 m s^{-1} and the speed of Q is 1.5 m…

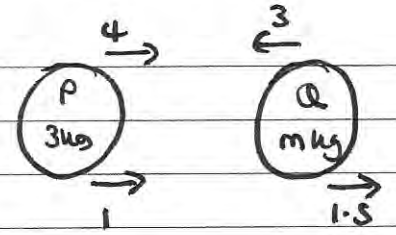
(a) Find the magnitude of the impulse exerted on P in the collision.

(3)

(b) Find the value of m.

(3)

a)



Mom P before = 12 ⇒ Impulse = 9Ns

Mom P after = 3

b) CLM total Mom before = total mom after

$12 + -3m = 3 + 1.5m \Rightarrow 9 = 4.5m$

$\therefore m = 2kg$

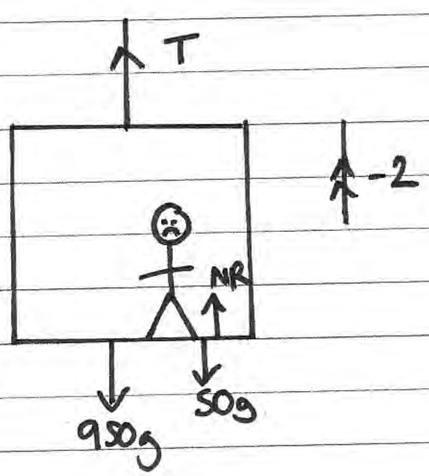
2. A woman travels in a lift. The mass of the woman is 50 kg and the mass of the lift is 950 kg. The lift is being raised vertically by a vertical cable which is attached to the top of the lift. The lift is moving upwards and has constant deceleration of 2 m s^{-2}. By modelling the cable as being light and inextensible, find

(a) the tension in the cable,

(3)

(b) the magnitude of the force exerted on the woman by the floor of the lift.

(3)



RF↑ = ma

$T - 1000g = 1000 \times -2$

$T - 9800 = -2000$

$\therefore T = 7800N$

b) RF↑ = ma ⇒ $NR - 50g = 50 \times -2$ ⇒ NR = 390N

3.

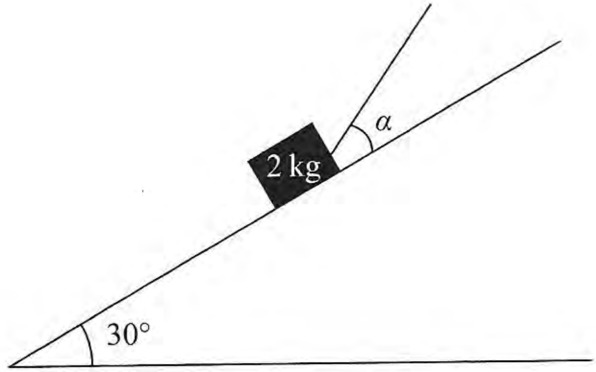
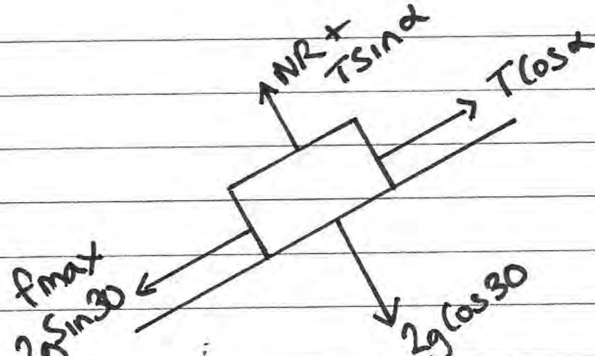
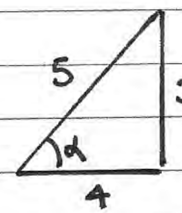


Figure 1

A box of mass 2 kg is held in equilibrium on a fixed rough inclined plane by a rope. The rope lies in a vertical plane containing a line of greatest slope of the inclined plane. The rope is inclined to the plane at an angle α, where $\tan \alpha = \frac{3}{4}$, and the plane is at an angle of 30° to the horizontal, as shown in Figure 1. The coefficient of friction between the box and the inclined plane is $\frac{1}{3}$ and the box is on the point of slipping up the plane. By modelling the box as a particle and the rope as a light inextensible string, find the tension in the rope.

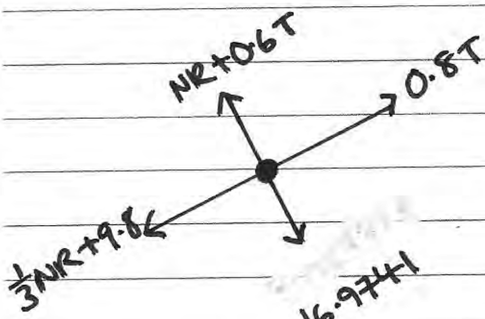
(8)





$\sin\alpha = \frac{3}{5}$

$\cos\alpha = \frac{4}{5}$



$R_f \uparrow = 0 \Rightarrow NR = 16.9741 - 0.6T$

$R_f = 0 \Rightarrow 0.8T = \frac{1}{3}NR + 9.8$

$\Rightarrow 0.8T = 5.65803 - 0.2T + 9.8$

$\therefore \; T = 15.5N \; (3sf).$

4. A lorry is moving along a straight horizontal road with constant acc[illegible]n. T[illegible] passes a point A with speed u m s^{-1}, ($u < 34$), and 10 seconds later passes a p[illegible] 34 m s^{-1}. Given that $AB = 240$ m, find

(a) the value of u,

(b) the time taken for the lorry to move from A to the mid-point of AB.

(6)

a) $s = 240$ $\quad s = \frac{(u+v)t}{2} \Rightarrow 240 = \frac{(u+34)\times 10}{2}$

$u = u$

$v = 34$

$a =$ $\quad \Rightarrow u + 34 = 48 \quad \therefore \underline{u = 14}$

$t = 10$

b) $s = 120$ $\quad$ from a) $v = u + at$

$u = 14$ $\quad 34 = 14 + 10a$

$v =$ $\quad 20 = 10a \Rightarrow \underline{a = 2}$

$a = 2$

$t =$

$s = ut + \frac{1}{2}at^2 \Rightarrow 120 = 14t + \frac{1}{2}(2)t^2$

$\Rightarrow t^2 + 14t - 120 = 0$

$\Rightarrow (t+20)(t-6) = 0$

$\therefore \underline{t = 6 \text{ sec}}$

5. A car is travelling along a straight horizontal road. The car takes 120 s to travel b two sets of traffic lights which are 2145 m apart. The car starts from rest at the first s traffic lights and moves with constant acceleration for 30 s until its speed is 22 m s^{-1}. Th car maintains this speed for T seconds. The car then moves with constant deceleration, coming to rest at the second set of traffic lights.

(a) Sketch, in the space below, a speed-time graph for the motion of the car between the two sets of traffic lights.

(2)

(b) Find the value of T.

(3)

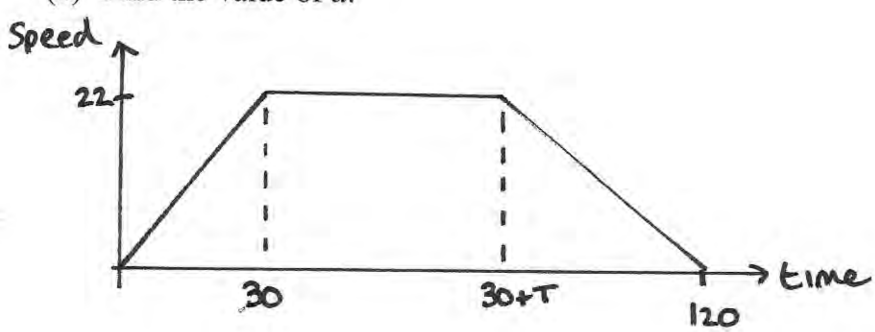
A motorcycle leaves the first set of traffic lights 10 s after the car has left the first set of traffic lights. The motorcycle moves from rest with constant acceleration, a m s^{-2}, and passes the car at the point A which is 990 m from the first set of traffic lights. When the motorcycle passes the car, the car is moving with speed 22 m s^{-1}.

(c) Find the time it takes for the motorcycle to move from the first set of traffic lights to the point A.

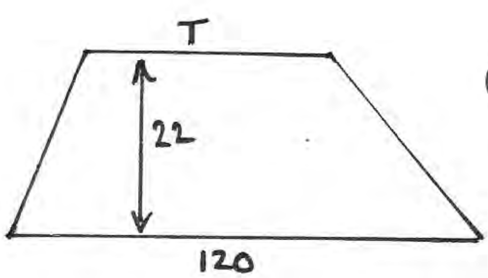
(4)

(d) Find the value of a.

(2)



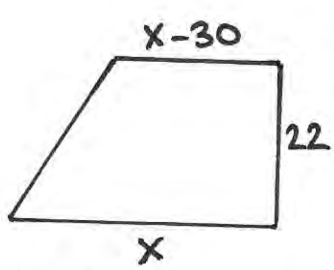
b)



$$\frac{(T+120)\times 22}{2} = 2145$$

$T+120 = 195 \quad \therefore T = \underline{75\text{sec}}$

c)



$$\frac{(x+x-30)\times 22}{2} = 990$$

$\Rightarrow 2x-30 = 90 \quad \Rightarrow x = 60$ sec.

$\therefore$ The bike takes 50 sec.

d) $s=990$
$u=0$
$t=50$

$s = ut + \frac{1}{2}at^2 \Rightarrow 990 = \frac{1}{2}(a)\times 50^2$

$\therefore a = \underline{0.792}$

6. A beam AB has length 15 m. The beam rests horizontally in equilibrium on two small supports at the points P and Q, where $AP = 2$ m and $QB = 3$ m. When a child of mass 50 kg stands on the beam at A, the beam remains in equilibrium and is on the point of tilting about P. When the same child of mass 50 kg stands on the beam at B, the beam remains in equilibrium and is on the point of tilting about Q. The child is modelled as a particle and the beam is modelled as a non-uniform rod.

(a) (i) Find the mass of the beam.

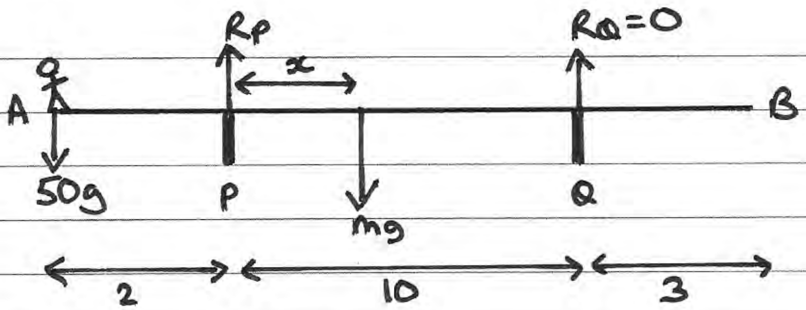
(ii) Find the distance of the centre of mass of the beam from A.

(8)

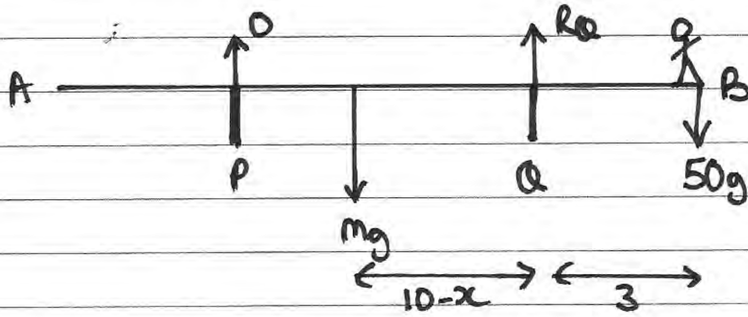
When the child stands at the point X on the beam, it remains horizontal and in equilibrium. Given that the reactions at the two supports are equal in magnitude,

(b) find AX.

(6)



P: $50g \times 2 = mg \times x \Rightarrow 100 = mx$



Q: $mg(10-x) = 50g \times 3 \Rightarrow 10m - mx = 150$

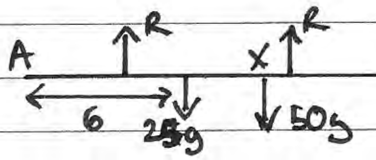
$\Rightarrow 10m - 100 = 150$

$10m = 250 \quad \therefore m = 25kg.$

ii) $100 = 25x \Rightarrow x = 4$

$\therefore$ C.O.M $= 6m$ from A.

b)



$R(\uparrow) = 0 \Rightarrow 2R = 75g \Rightarrow R = 37.5g$

A: $R\times 2 + R\times 12 = 25g\times 6 + 50g\times AX$

$525g = 150g + 50g \times AX \quad \therefore AX = 7.5m$

7. *[In this question, the horizontal unit vectors* **i** *and* **j** *are directed due ... respectively.]*

The velocity, **v** m s^{-1}, of a particle P at time t seconds is given by

$$\mathbf{v} = (1 - 2t)\mathbf{i} + (3t - 3)\mathbf{j}$$

(a) Find the speed of P when $t = 0$

(3)

(b) Find the bearing on which P is moving when $t = 2$

(2)

(c) Find the value of t when P is moving

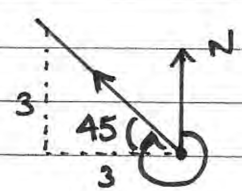
(i) parallel to **j**,

(ii) parallel to $(-\mathbf{i} - 3\mathbf{j})$.

(6)

a) $v = i - 3j$ speed $= \sqrt{1^2 + 3^2} = 3.16 \text{ ms}^{-1}$ (3sf)

b) $t = 2$ $v = -3i + 3j$



$\therefore$ bearing $= 315°$

c) i) i component $= 0$ $\Rightarrow t = \frac{1}{2}$

ii) $\begin{pmatrix} 1-2t \\ 3t-3 \end{pmatrix} = \lambda \begin{pmatrix} -1 \\ -3 \end{pmatrix} \Rightarrow$ $1 - 2t = -\lambda \Rightarrow \lambda = 2t - 1$

$3t - 3 = -3\lambda$ $\lambda = 1 - t$

$\Rightarrow 2t - 1 = 1 - t \Rightarrow 3t = 2 \Rightarrow t = \frac{2}{3}$

8.

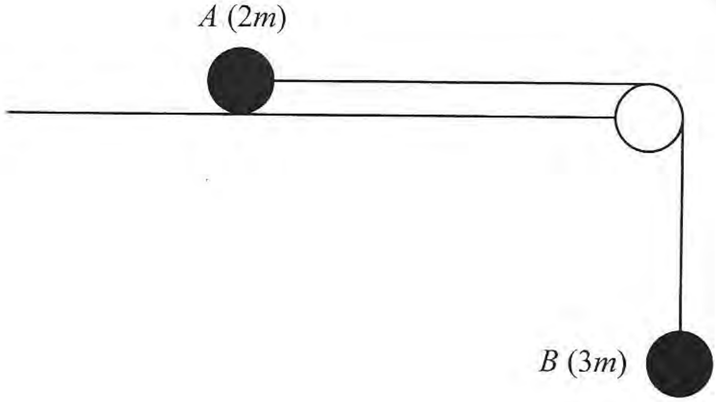


Figure 2

Two particles A and B have masses $2m$ and $3m$ respectively. The particles are attached to the ends of a light inextensible string. Particle A is held at rest on a smooth horizontal table. The string passes over a small smooth pulley which is fixed at the edge of the table. Particle B hangs at rest vertically below the pulley with the string taut, as shown in Figure 2. Particle A is released from rest. Assuming that A has not reached the pulley, find

(a) the acceleration of B,

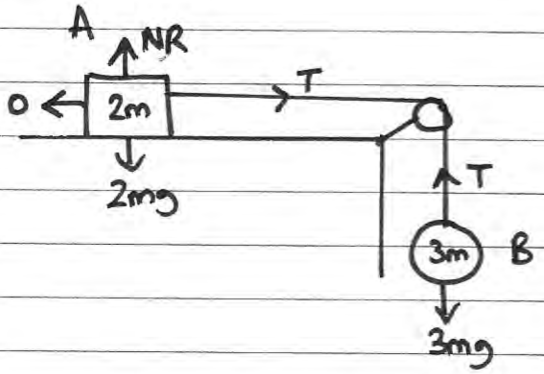
(5)

(b) the tension in the string,

(1)

(c) the magnitude and direction of the force exerted on the pulley by the string.

(4)



$\overrightarrow{RF} = ma$

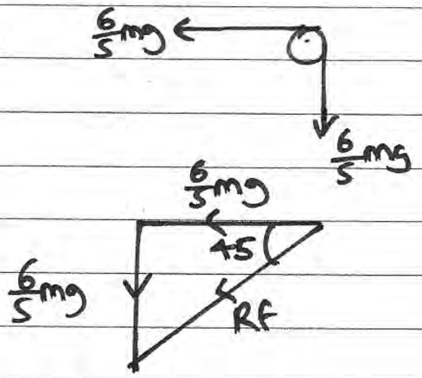
Ⓐ $T = 2ma$

Ⓑ $3mg - T = 3ma$ $+$

$3\cancel{m}g = 5\cancel{m}a \quad \therefore a = \frac{3}{5}g$

b) $T = \frac{6}{5}mg$

c)



$RF^2 = \left(\frac{6}{5}mg\right)^2 + \left(\frac{6}{5}mg\right)^2$

$\therefore RF = \frac{6\sqrt{2}}{5}mg$

acting $45°$ below horizontal.